6.7 Videos Guide

6.7a

Definitions: (hyperbolic functions)

- $\sinh x = \frac{e^x - e^{-x}}{2}$ $\qquad$ $\operatorname{csch} x = \frac{1}{\sinh x}$
- $\cosh x = \frac{e^x + e^{-x}}{2}$ $\qquad$ $\operatorname{sech} x = \frac{1}{\cosh x}$
- $\tanh x = \frac{\sinh x}{\cosh x}$ $\qquad$ $\coth x = \frac{\cosh x}{\sinh x}$

6.7b

Exercises:

- Find the numerical value of each expression.
  a) $\sinh 4$ $\qquad$ b) $\sinh(\ln 4)$
- Prove the identity.
  $\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$

6.7c

- Derivatives of hyperbolic functions
  - $\frac{d}{dx}(\sinh x) = \cosh x$
  - $\frac{d}{dx}(\cosh x) = \sinh x$
  - $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$
  - $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$
  - $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$
  - $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$
  - Note that reversing these gives integration rules

Exercises:

- Find the derivative. Simplify where possible.
  $F(t) = \ln(sinh\, t)$
- Evaluate the integral.
  $$\int \frac{\operatorname{sech}^2 x}{2 + \tanh x}\, dx$$

6.7d

- Inverse hyperbolic functions
  - $\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right),$ $\qquad$ $x \in \mathbb{R}$
  - $\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right),$ $\qquad$ $x \geq 1$
  - $\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right),$ $\qquad$ $-1 < x < 1$

- Derivatives of inverse hyperbolic functions
  - $\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$ $\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$
  - $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$ $\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$
  - $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$, $|x| < 1$ $\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$, $|x| > 1$

Exercise:

- Evaluate the integral.

$$\int_0^1 \frac{1}{\sqrt{16t^2+1}}\, dt$$